First-Principles Modeling of Wireless Networks for Rate Control

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***Abstract*—Achieving fair and optimal data rates in wireless networks is an area of continued research. Distributed algorithms have been developed directly from mathematical optimization problems that guarantee fair and optimal rates. However, the algorithms developed thus far are based on simplified models of wireless networks. This research presents a first-principles model of wireless networks that reduces to the classical models under certain limiting conditions. The model uses random sets to represent the times during which a channel is perceived to be utilized. Although the resulting optimization problem is non-convex, its solution can be derived offline to offer insight into situations where the classical models succeed or fail. We provide the framework for a branch and bound solution to this offline problem.**

I. Introduction

Wireless networks are often used as a low-cost alternative to wired infrastructures, while also accommodating mobile users. The most prevalent medium access control (MAC) protocol used in wireless networks is defined in the IEEE 802.11 standard. However, research has shown that when the wireless network is extended to multiple hops the 802.11 MAC is plagued with serious fairness and efficiency problems, sometimes completely starving one data flow in favor of another [1]. This is in part due to the fact that sharing resources in a wireless network is a fundamentally different problem than in a wired network and requires some theoretical understanding to solve.

As a result of these problems, rate allocation in wireless networks to achieve maximum network utilization and fairness has become a popular area of research. Seminal research in this area includes [2], [3] for wired networks, which has been easily extended to wireless networks in [4], [5], [6], [7], [8], [9], [10]. See [11] for a survey on this and other active research topics for wireless networks.

This research attempts to answer a fundamental question: *Given a wireless network topology and a set of active data flows between source nodes and destination nodes, what allocation of rates is optimal and fair?* In the network utility maximization (NUM) approach, an objective function for the network is defined, typically a sum of utility functions for each link's or flow's sending rate, where the form of the utility function defines a particular notion of fairness. Next, a set of constraints is used to model the unique characteristics of the wireless network, such as carrier sensing or interference constraints. The solution to this optimization problem will then yield a set of rates that maximize network utility for the links or flows. These rates can then be used as input to a rate controller that sits on top of (or inside) the MAC protocol, limiting the packet transmission rate for each flow or link. When the optimization problem is convex, it can often be translated into a distributed rate control algorithm, making it practical to deploy in a wireless network.

Our focus is on the constraints used to model the wireless network, as this is the critical piece in the NUM approach. If the model is inaccurate, then the optimization problem may not yield an accurate or optimal solution. We limit our study to stationary, multi-hop wireless networks that use CSMA, such as the 802.11 MAC. This broadly characterizes the most widely-used wireless networks in the field, often referred to as mesh networks.

This paper develops a first-principles model of wireless networks for the rate control problem. By first-principles, we mean that the most basic assumptions are made of how multi-hop wireless networks with CSMA operate. In this model, perceived times that the medium is occupied are represented as a random set. Our model may also be classified as a measurement based model, as it takes as inputs the probabilities of links carrier sensing or interfering with each other. Such an approach is more realistic than physical layer modeling, such as the various signal fading models with SINR thresholds, and has been used significantly to model wireless networks for various purposes [12], [13], [14], [15], [16], [17], [9]. A combination of measurement based modeling and physical layer modeling is used in [14] to determine probabilities of carrier sensing and interference between pairs of nodes. Kashyap, et al [16] extends this idea to also model probabilities of carrier sensing and interference of groups of nodes in order to determine the capacity of a wireless link. We note specifically that their approach is very similar to ours, modeling the states (transmitting, deferring, idle) of a node as random sets. However, they also model specific aspects of the 802.11 MAC as opposed to generic CSMA, and their model predicts an uncontrolled environment, where there is no rate controller other than the 802.11 MAC. In the future, we hope to combine the qualities of ours and Kashyap's models to achieve further accuracy for rate control problems.

In this work, we show that, under limiting conditions, our model reduces to previously proposed models in the literature.

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s_i	Sending rate of link i.
r_i	Receiving rate of link i.
d_i	Delivery ratio of link i.
a_{ij}	Receiving interference probability of link j interfering with link i.
L	Set of links.
L_i	Set of all links in L except i.
C	Set of maximal cliques.
$C(i)$	Set of maximal cliques containing link i.
$L(j)$	Set of links in maximal clique j.

TABLE I
NOTATION USED IN THE MAXIMAL CLIQUE MODEL AND THE PARTIAL INTERFERENCE MODEL.

Specifically, with the assumption of binary, symmetric sensing, our model reduces to the common maximal clique model used by seminal research in this area [4], [8]. Likewise, with the assumption of no carrier sensing between interfering links, our model reduces to the partial interference model we previously developed [10]. We conclude by showing that our model induces an optimization problem and scoring mechanism that can be used to compare the performance of various rate controllers, should our model indeed prove to be more accurate. In this case, solving the network utility maximization problem using the first-principles model will provide an upper bound on the performance of all rate control policies.

II. Classical Models

In this section we review two classical models of wireless networks that have been used to solve the fair rate control problem: the *maximal clique model*, which formulates constraints on the sending rates, and the *partial interference model*, which supplements the maximal clique model by formulating constraints on the receiving rates. Table I presents the notation used in these models.

A. The Maximal Clique Model

The maximal clique model is the most widely used model for rate optimization in wireless networks [4], [8]. Figure 1 shows how a contention graph is inferred from a wireless network. This graph has a vertex representing each active link, and each edge signifies that two links *contend*, or cannot send at the same time. For each maximal clique j, its links' sending rates s must sum to at most some clique capacity, which for our purposes will always be 1:

$$\sum_{i \in L(j)} s_i \leq 1, \quad \forall j \in C. \tag{1}$$

B. The Partial Interference Model

The partial interference model supplements the maximal clique model with constraints on the receiving rates [10]. The model is based on an empirical study of carrier sensing and interference in a wireless mesh network [17]. The constraint on each receiving rate is

$$r_i = d_i s_i \prod_{j \in L_i} (1 - a_{ij} s_j), \quad \forall i \in L, \tag{2}$$

where the delivery ratio d_i implies inherent loss over the link.

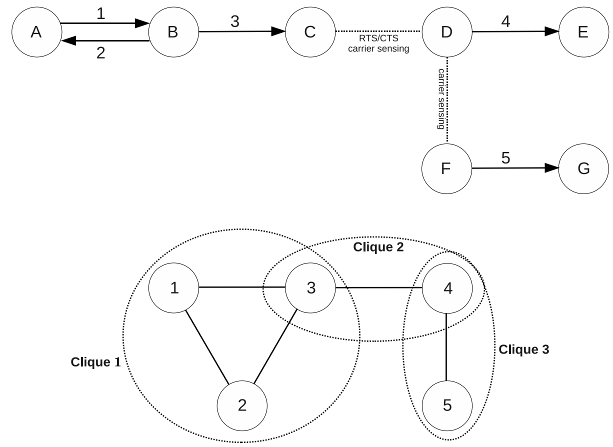

Fig. 1. An example wireless network and its corresponding contention graph with the maximal cliques circled. Links 1, 2, and 3 contend because they share node B. Links 4 and 5 contend because sending nodes D and F are within carrier sensing range of each other. Assuming RTS/CTS is enabled, links 3 and 4 contend because sending node D is within carrier sensing range of receiving node C, which sends out CTS signals. This model infers that links in a maximal clique have sending rates summing to at most one.

c_{ij}	Carrier sensing probability of link j being sensed by link i.
S_i	Effective sending rate of all other links as observed by link i.
R_i	Effective (receiving) interference rate at link i due to all other links.
K_i	Set of links that contend with link i.
$p_1 \backslash p_2$	Set of elements in p_1 but not in p_2.
$\mathcal{P}(p)$	Set of all subsets of p except the empty set.
$\mathcal{P}_z(p)$	Set of all subsets of p with $\lvert p \rvert = z$.

TABLE II
ADDITIONAL NOTATION USED IN THE FIRST-PRINCIPLES MODEL.

III. The First-Principles Model

The design of the classical models raises some questions. How do we know that mutually contending links (maximal cliques) implies that their rates must sum to at most 1? How can the maximal clique model be logically extended to consider the case of partial carrier sensing, where there is a continuous range of probabilities that nodes can sense each other, and the notion of cliques is immediately destroyed? Why, in the partial interference model, is the effect of each interfering link multiplicative? We seek to answer these questions by developing a model from a more theoretical standpoint by using random sets to represent observed times the medium is occupied. The additional notation is given in Table II.

The following elementary assumptions are made:

- *Discretization of time.* Time is divided into large blocks that are further divided into equally sized slots. During each time slot, each link is either sending or not sending.
- *Uniform random selection.* For each time block T, each link has a set $F \subset T$ of available time slots in which to send, and a set $X \subset F$ when it does send. Each $t \in F$ has an equal probability of being in X.
- *Negligible indirect scheduling.* If link i senses links j

and k sending during $X_j, X_k \subset T$, respectively, then dependencies of $X_j \cup X_k$ on the sending times of any link $l \neq i, j, k$ are negligible.

When considering the effective rate of links j and k as perceived by link i, realistically there may be some other link l (or even a set of other links) that cause the rates of j and k to overlap more or less than usual, which could in turn affect how much link i can send. The last assumption simply states that these effects are negligible. We recognize that it can significantly impact the accuracy of the model. This concern will be addressed in future research with empirical testing.

A. Uniform Random Sets

A link's sending rate is restricted by how much it senses that others are sending, in other words, their effective rate. Our goal is to find a formula for this effective rate by calculating the union of each random set representing a link's sending times. We begin by defining a uniform random set:

Definition 1. *Let T and $F \subset T$ be finite sets. Also let $\xi_t : \Omega \to \{0,1\}$ for all $t \in T$ be a collection of i.i.d. random variables on the probability space $(\Omega, \mathcal{A}, P)$. Then $X : \Omega \to 2^T$, where*

$$X(\omega) = \{t \in F : \xi_t(\omega) = 1\}, \tag{3}$$

is a uniform random set on F. Moreover, if F is a random set, then

$$X(\omega) = \{t \in F(\omega) : \xi_t(\omega) = 1\} \tag{4}$$

also defines a uniform random set on F. The set F is called the parent of X.

The deterministic set T can be considered as the entire time block. If link i sends at rate s_i, link j hears $c_{ji}s_i$ of T being occupied. Thus the $c_{ij}s_j$ that link i hears must be a random set chosen from a subset of T, namely during which j did not hear i. For this reason we include in the definition of a uniform random set a parent set, or free space F, to which it is restricted.

Now consider another link k adding to the effective rate that i senses. The effective rates of j and k by themselves overlap in the total effective rate a certain amount depending on how much they sense each other. In this sense, the above definition still does not fully explain the interaction of uniform random sets. We thus define the independence of uniform random sets:

Definition 2. *Let $\mathbb{X}$ be a collection of uniform random sets with parents $\mathbb{F}$, enumerated by $N = \{1, \ldots, n\}$. Then their independence is*

$$h(N) = E_{t\in\cap\mathbb{F}}\left[\frac{\Pr(t \in \cap\mathbb{X})}{\prod_{i\in N}\Pr(t \in X_i \cap \mathbb{F})}\right]. \tag{5}$$

Let $|\cdot|$ denote the expected size of a random set. Averaging over all t in $\cap\mathbb{F}$, we have from (5) that

$$\Pr(t \in \cap\mathbb{X}) = h(N)\prod_{i\in N}\Pr(t \in X_i \cap \mathbb{F}),$$
$$\frac{|\cap\mathbb{X}|}{|\cap\mathbb{F}|} = h(N)\prod_{i\in N}\frac{|X_i \cap \mathbb{F}|}{|\cap\mathbb{F}|}.$$

But $|X_i \cap \mathbb{F}|$ is found by multiplying $|X_i|$ by the probability that an element in F_i is also in $\cap\mathbb{F}$:

$$|X_i \cap \mathbb{F}| = \frac{|\cap\mathbb{F}|}{|F_i|}|X_i|$$

so that

$$|\cap\mathbb{X}| = h(N)|\cap\mathbb{F}|\prod_{i\in N}\frac{|X_i|}{|F_i|}. \tag{6}$$

We can now invoke the inclusion-exclusion principle

$$\left|\bigcup_{X\in\mathbb{X}} X\right| = \sum_{p\in\mathcal{P}(\mathbb{X})}(-1)^{|p|-1}\left|\bigcap_{X\in p} X\right| \tag{7}$$

to find the size of the union of many uniform random sets.

Theorem 1. *Let $\mathbb{X}$ be a collection of uniform random sets with parents $\mathbb{F}$, enumerated by $N = \{1, \ldots, n\}$. Then*

$$|\cup\mathbb{X}| = \sum_{p\in\mathcal{P}(N)}(-1)^{|p|-1}\left(\prod_{i\in p}|X_i|\right)\frac{\left|\bigcap_{i\in p}F_i\right|}{\prod_{i\in p}|F_i|}h(p). \tag{8}$$

Proof: The result follows from (6) and (7). ∎

B. Derivation of the First-Principles Model

The sending constraint is given by

$$s_i + S_i \leq 1, \ \forall i \in L, \tag{9}$$

where

$$S_i = \sum_{p\in\mathcal{P}(L_i)}(-1)^{|p|-1}f_i(p)g_i(p)h(p), \tag{10}$$

$$f_i(p) = \prod_{j\in p}c_{ij}s_j, \tag{11}$$

$$g_i(p) = \frac{\phi_i(p)}{\prod_{j\in p}\phi_i(j)}, \tag{12}$$

$$\phi_i(p) = 1 - s_i\sum_{p'\in\mathcal{P}(p)}(-1)^{|p'|-1}\prod_{j\in p'}c_{ji}, \tag{13}$$

and the independence is given by

$$h(p) = \prod_{\{i,j\}\in\mathcal{P}_2(p)}(1 - c_{ij} - c_{ji} + c_{ij}c_{ji}). \tag{14}$$

The receiving constraint is given by

$$r_i = d_i(1 - R_i)s_i, \tag{15}$$

where

$$R_i = \sum_{p\in\mathcal{P}(L_i)}(-1)^{|p|-1}f'_i(p)h(p) \tag{16}$$

and

$$f'_i(p) = \prod_{j\in p}a_{ij}s_j. \tag{17}$$

Note that (14) is an approximation of (5). If one link carrier senses another link completely ($c_{ij} = 1$) then their random sets do not intersect. If any two random sets in p do not intersect, then the intersection of p is empty, which means that $h(p)$

should equal zero. Only when all random sets are independent should it equal one.

The formulas for S_i and R_i follow immediately from Theorem 1. In the case of R_i, since there is no intermediary correlating link for the interferers, F is the entire space with size 1. However, for S_i, we need to derive $g_i(p)$, which corresponds to the term of free spaces F in the theorem.

Link j observes a free space F_j of size $1 - c_{ji}s_i$, in which link i observes an occupied space X_j of size $c_{ij}s_j$. The free space F_j consists of a portion during which link i is not sending, denoted Γ, and a portion during which link i is sending but not heard by j, denoted Ψ_j. Their sizes are given by

$$|\Gamma| = 1 - s_i$$

and

$$|\Psi_j| = (1 - c_{ji})s_i.$$

Let $F(p) = \cap_{j\in p} F_j$ and $\Psi(p) = \cap_{j\in p}\Psi_j$. Then, since every Ψ_j is an independent uniform random set within a common space of size s_i, we have by way of (6),

$$\begin{aligned} |\Psi(p)| &= \frac{\prod_{j\in p} |\Psi_j|}{s_i^{|p|-1}} \\ &= s_i \prod_{j\in p}(1 - c_{ji}). \end{aligned}$$

Thus

$$\begin{aligned} \phi_i(p) := |F(p)| = |\Gamma| + |\Psi(p)| \\ &= 1 - s_i + s_i \prod_{j\in p}(1 - c_{ji}) \\ &= 1 - s_i \sum_{p'\in\mathcal{P}(p)} (-1)^{|p'|-1} \prod_{j\in p'} c_{ji}. \end{aligned}$$

We call this function the transparency of i to p, and use it to simplify the notation in $g_i(p)$.

As a side note, it is straightforward to incorporate flow constraints into the model as well, by introducing mappings $t(\cdot)$ from the hop number in the flow to the index of the link considered. Then

$$s_{t(m)} \le r_{t(m-1)} \tag{18}$$

for each hop m and each flow mapping t ensures that no subsequent hop sends more than it receives.

IV. Reduction to the Classical Models

A. Reduction to the Maximal Clique Model

The first-principles model reduces to the maximal clique model when carrier sensing is binary and symmetric. We prove this by showing that the set of feasible sending rates in one model is equivalent to the feasible set in the other model.

We first formally define the two sets of feasible rates based on each model.

Definition 3. *Given a contention graph (L, C), the set $\mathcal{S}_1$ consists of all vectors of sending rates s that satisfy* (1).

Definition 4. *Given a contention graph (L, C), the set $\mathcal{S}_2$ consists of all vectors of sending rates s that satisfy* (9)*, where*

$$S_i = \sum_{p\in\mathcal{P}(K_i)} \left(\frac{-1}{1 - s_i}\right)^{|p|-1} h(p) \prod_{j\in p} s_j$$

and

$$h(p) = \begin{cases} 0, & \exists i, j \in p : i \in K_j, \\ 1, & \text{otherwise.} \end{cases}$$

Theorem 2. *Given a contention graph (L, C), $\mathcal{S}_1 = \mathcal{S}_2$.*

Proof: First, note that $\mathcal{S}_1$ is equivalent to the set of s satisfying

$$s_i + \max\{S_i(j) : j \in C(i)\} \le 1, \ \forall i \in L,$$

where $S_i(j) = \sum_{l\in L(j)\setminus i} s_l$. To show that $\mathcal{S}_1 = \mathcal{S}_2$, it suffices to show that, for any $i \in L$, the constraint boundary is equivalent in $\mathcal{S}_1$ and $\mathcal{S}_2$. Thus, letting

$$s_i = 1 - \max\{S_i(j) : j \in C(i)\},$$

we seek to show that $S_i = \max\{S_i(j) : j \in C(i)\}$.

Without loss of generality, let $i = 0$ and $K_0 = \{1, \ldots, n\}$, where $L(1) = \{1, \ldots, m\}$ is the most constraining maximal clique and $S_0(1) = \sum_{j=1}^m s_j$. Then

$$\begin{aligned} S_0 &= \sum_{p\in\mathcal{P}(K_0)} \left(\frac{-1}{S_0(1)}\right)^{|p|-1} h(p) \prod_{j\in p} s_j \\ &= \sum_{z=1}^{n} \left(\sum_{p\in\mathcal{P}_z(K_0)} \left(\frac{-1}{S_0(1)}\right)^{z-1} h(p) \prod_{j\in p} s_j \right) \\ &= \sum_{j=1}^{n} s_j + \sum_{z=2}^{n} \left(\frac{-1}{S_0(1)}\right)^{z-1} \left(\sum_{p\in\mathcal{P}_z(K_0)} h(p) \prod_{j\in p} s_j \right) \\ &= S_0(1) + \sum_{j=m+1}^{n} s_j \\ &\quad + \sum_{z=2}^{n} \left(\frac{-1}{S_0(1)}\right)^{z-1} \left(\sum_{p\in\mathcal{P}_z(K_0)} h(p) \prod_{j\in p} s_j \right). \end{aligned}$$

We therefore must determine that

$$\sum_{j=m+1}^{n} s_j + \sum_{z=2}^{n} \left(\frac{-1}{S_0(1)}\right)^{z-1} \left(\sum_{p\in\mathcal{P}_z(K_0)} h(p) \prod_{j\in p} s_j \right) = 0. \tag{19}$$

Note that $h(p) = 0$ for any p that has at least two elements from $L(1)\setminus\{0\}$, so that

$$\begin{aligned} \sum_{p\in\mathcal{P}_z(K_0)} h(p) \prod_{j\in p} s_j = S_0(1) \sum_{p\in\mathcal{P}_{z-1}(K_0\setminus L(1))} h(p) \prod_{j\in p} s_j \\ + \sum_{p\in\mathcal{P}_z(K_0\setminus L(1))} h(p) \prod_{j\in p} s_j. \end{aligned}$$

This is substituted into the argument of the second sum of (19) to obtain

$$\left(\frac{-1}{S_0(1)}\right)^{z-1} \sum_{p\in\mathcal{P}_z(K_0)} h(p)\prod_{j\in p} s_j = - \frac{(-1)^{z-2}}{S_0(1)^{z-2}} \sum_{p\in\mathcal{P}_{z-1}(K_0\backslash L(1))} h(p)\prod_{j\in p} s_j + \frac{(-1)^{z-1}}{S_0(1)^{z-1}} \sum_{p\in\mathcal{P}_z(K_0\backslash L(1))} h(p)\prod_{j\in p} s_j.$$

The second term above for some z cancels with the first term in the corresponding $z+1$ equation, and the first term for $z=2$ cancels with $\sum_{j=m+1}^{n} s_j$. We need only check that the second term for $z=n$ goes to zero. By inspection,

$$\sum_{p\in\mathcal{P}_n(K_0\backslash L(1))} h(p)\prod_{j\in p} s_j = 0,$$

because $|K_0\backslash L(1)| < n$. Thus, $S_0 = S_0(1)$ and $\mathcal{S}_1 = \mathcal{S}_2$. ■

B. Reduction to the Partial Interference Model

We now show that when interferers of link i do not contend with each other, (15) and (16) from the first-principles model reduce to (2) in the partial interference model. To do this, we present a simple arithmetical theorem:

Theorem 3. *For some set L of indices,*

$$\prod_{j\in L}(1-x_j) = 1 - \sum_{p\in\mathcal{P}(L)} (-1)^{|p|-1}\prod_{j\in p} x_j. \tag{20}$$

Proof: Without loss of generality, let $L = \{1,\ldots,n\}$. We prove by induction. The base case $n=1$ holds trivially. Assuming (20) holds for n, we need to show that it holds for $n+1$. Defining $N = L\backslash(n+1)$,

$$\begin{aligned}\prod_{j\in L}(1-x_j) &= (1-x_{n+1})\prod_{j\in N}(1-x_j)\\ &= (1-x_{n+1})\left(1-\sum_{p\in\mathcal{P}(N)}(-1)^{|p|-1}\prod_{j\in p}x_j\right)\\ &= 1-\sum_{p\in\mathcal{P}(N)}(-1)^{|p|-1}\prod_{j\in p}x_j\\ &\quad - x_{n+1} + \sum_{p\in\mathcal{P}(N)}(-1)^{|p|-1}x_{n+1}\prod_{j\in p}x_j\\ &= 1-\sum_{p\in\mathcal{P}(L)}(-1)^{|p|-1}\prod_{j\in p}x_j.\end{aligned}$$

■

Applying this result to (2) is straightforward, replacing x_j with $a_{ij}s_j$. Since under the limiting condition of no contending interferers we have $h(p)=1$, we see that (15) does indeed reduce to (2).

V. Solution to the Optimization Problem

Assuming that d is always unity, the first-principles NUM problem is

$$\begin{aligned}\mathbf{P}: \quad &\text{maximize} && \sum_{i\in L}\ln r_i\\ &\text{subject to} && r_i = (1-R_i)s_i, \quad \forall i\in L,\\ &&& s_i + S_i \le 1, \quad \forall i \in L,\end{aligned} \tag{21}$$

where we have chosen the natural logarithm as the utility function, enforcing proportional fairness. We do not show it here, but instances of this problem are frequently non-convex. A branch and bound solution solves the problem by successively dividing the hypercube in which the feasible set resides into smaller regions, and evaluating lower and upper bound functions for the optimal value in each region. The bounds on each region allow one to conclude that some regions need not be divided further. A good tutorial on branch and bound appears in [18].

To implement branch and bound, we need only develop efficient upper and lower bound functions for each sub-problem of **P**. Let $\mathbf{P}_k$ be the k-th sub-problem of **P** in the algorithm. A standard interior point solver Φ operating on $\mathbf{P}_k$ is sufficient to get a lower bound and the corresponding feasible point. To get an upper bound, we need to formulate a new, convex problem $\mathbf{P}'$ such that its solution is an upper bound to the solution of **P**.

To begin, we wish to replace the sending constraint in **P** with something that is convex and will enclose the old constraints. This is done by replacing the effective rate S_i with something smaller, since this will leave more room for s_i to increase. Because S_i is the size of the union of several sets with sizes $c_{ij}s_j$, it follows that $S_i \ge \max c_{ij}s_j$. Finally the new constraint

$$s_i + \max_{j\in L_i} c_{ij}s_j \le 1$$

is equivalent to the family of constraints

$$s_i + c_{ij}s_j \le 1, \quad \forall j \in L_i,$$

which are all linear.

Now we must modify the receiving constraint. First, note that replacing it with

$$r_i \le (1-R_i)s_i$$

does not change the solution to the problem.We next introduce the variable

$$y_i = r_i/s_i$$

so that the inequality becomes

$$y_i + R_i \le 1.$$

Then, R_i is replaced in the same manner that S_i was replaced, which yields the family of constraints

$$y_i + a_{ij}s_j \le 1, \quad \forall j \in L_i.$$

The change of variables from (s, r) to (s, y) also modifies the appearance of the objective function to

$$\sum_{i \in L} (\ln s_i + \ln y_i) \,.$$

The new, convex problem that bounds $\mathbf{P}$ is

$$\begin{aligned} \mathbf{P}': \quad & \text{maximize} && \sum_{i \in L} (\ln s_i + \ln y_i) \\ & \text{subject to} && y_i + a_{ij} s_j \leq 1, \qquad \forall i, j \in L,\ i \neq j, \\ & && s_i + c_{ij} s_j \leq 1, \qquad \forall i, j \in L,\ i \neq j. \end{aligned} \tag{22}$$

Let $\mathbf{P}'_k$ be the k-th sub-problem of $\mathbf{P}'$ in the algorithm. Then Φ operating on $\mathbf{P}'_k$ is sufficient to get an upper bound. Thus the branch and bound method, supplemented with the bound functions of $\Phi(\mathbf{P}_k)$ and $\Phi(\mathbf{P}'_k)$, solves $\mathbf{P}$ with efficient computation at each step.

VI. Conclusion

This paper has analyzed the underlying models in optimization techniques for rate control in wireless networks, specifically, the maximal clique model and the partial interference model. A new model, called the first-principles model, was developed based on probability laws of random, overlapping signals. This model accounts for partial and asymmetric carrier sensing and receiving interference. It has been shown that when the first-principles model is limited to binary, symmetric sensing, it reduces to the maximal clique model; and when it is limited to no carrier sensing between interfering links, it reduces to the partial interference model.

The first-principles model still has its own limitations. When measuring the effective rate (or amount the medium is occupied) at a particular link i, the model accounts for indirect scheduling of links j and k through link i, but not through any other link. We plan to analyze the significance of this limitation in future research through empirical testing.

The optimization problem for rate control induced by this model is non-convex, and cannot readily be separated into a form allowing a distributed solution; however, despite its complexity, it is still useful in understanding the performance of rate controllers. If the model is an accurate representation of the behavior of real wireless networks, its associated optimization problem produces a tight upper bound on the performance of all controllers. An outline of a branch and bound algorithm to do this has been presented in this paper. Future work will implement this algorithm, and also run simulated and real test scenarios to compare various rate controllers.

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